ABSTRACT OF THE DISCLOSURE

Hierarchical analysis of Markov chain usage models is provided. In some embodiments elements of an expanded matrix are calculated as a function of parent elements and child elements, rather than by direct inversion techniques. The expanded matrix is a first moment matrix of a flattened model. In hierarchical models, the flattened model represents a child model instantiated within a parent model. The child elements are elements of a child matrix, which is a first moment matrix of the child model. The parent elements are elements of a parent matrix, which is a first moment matrix of the parent model.